Master's degree research project
Convex optimization approaches to sparse estimation over the simplex

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The **simplex** Δ_N is the set of vectors $\mathbf{x}$ in $\mathbb{R}^N$ with $x_n \geq 0$ and $\sum_{n=1}^{N} x_n = 1$. If $\mathbf{x} \in \Delta_N$, its elements x_n can be viewed as *proportions*. Therefore, the simplex appears in a wide field of scientific problems, including finance (portfolio optimization), genomics, signal and image processing. In **image processing**, we can mention two problems, which can be formulated as the minimization of a convex function under the constraint that the variables belong to the simplex: the linear unmixing problem in multispectral imaging [1] and segmentation [2].

Often, minimization of a convex function over the simplex yields a solution $\mathbf{x}$ with some degree of **sparsity**; that is, only a few elements x_n are nonzero. But in some applications, we want more sparsity: the number of nonzero elements of the solution should be very small, and even equal to 1 if $\mathbf{x}$ is an assignment vector, which indicates which element is optimal among a set of N possible ones. We cannot enforce sparsity directly in the problem formulation, because it would become nonconvex and very difficult to solve.

This research project aims at developing new **convex** formulations of sparse optimization over the simplex, implementing them in Matlab, and evaluating them on some test problems, e.g. in image processing. Projecting a vector onto the simplex is easy [3], but convex optimization over the simplex requires using *splitting* schemes, which are iterative algorithms with simple operations at every iteration [4].

The internee must have skills in convex optimization and Matlab (or Python) programming. He should be motivated to continue with a PhD after this internship.

[1] J. M. Bioucas-Dias, A. Plaza, N. Dobigeon, M. Parente, Q. Du, and J. Chanussot, "Hyperspectral unmixing overview: geometrical, statistical, and sparse regression-based approaches," *IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, 5(2):354–379, 2012.

[2] L. Condat et N. Pustelnik, "Segmentation d'image par optimisation proximale," *actes du GRETSI*, Lyon, 2015.

[3] L. Condat, "Fast Projection onto the Simplex and the l1 Ball," *Mathematical Programming Series A*, 2015.

[4] P. L. Combettes and J.-C. Pesquet, "Proximal splitting methods in signal processing," in *Fixed-Point Algorithms for Inverse Problems in Science and Engineering*, Springer-Verlag, 2010, pp. 185–212.

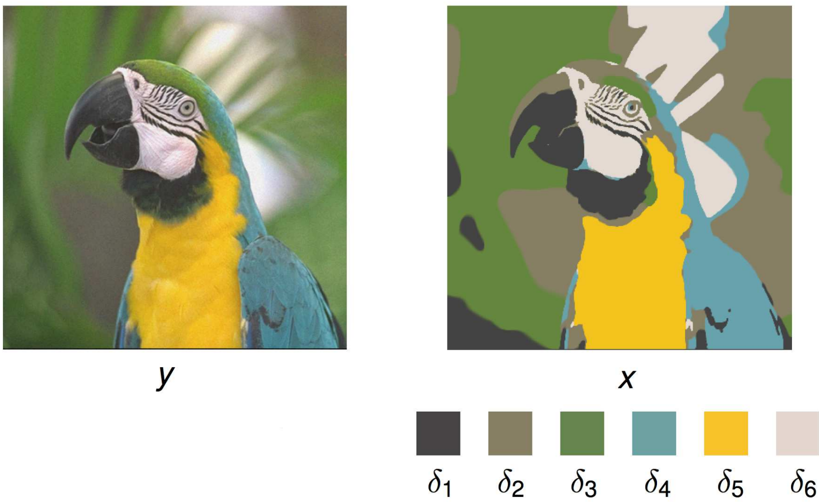

Figure 1: Image segmentation, more precisely piecewise constant approximation of an image y with N labels (colors) fixed beforehand ($N = 6$ in the example above, the labels are shown), can be performed by solving a convex optimization problem [2]. At every pixel, an assignment vector in Δ_N is computed, which determines which proportion of every label is assigned to the pixel. Most assignment vectors are binary, but not all, especially at the edges.