Certified roundoff error bounds using convex optimization

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Keywords precision tuning; roundoff error; numerical accuracy; floating-point arithmetic; semidefinite programming; polynomial sums of squares; generalized eigenvalue problems; computer proofs.

Context Roundoff errors cannot be avoided when implementing numerical programs with finite precision. For each computation step, the roundoff error is very small but the accumulation of such errors can grow very quickly. A workaround to this issue consists of relying on other software tools to verify programs and proofs, allowing to certify the roundoff error bounds. This verification ability is especially important if one wants to explore a range of potential representations, for instance in the world of FPGAs. It becomes challenging when the program does not employ solely linear operations as nonlinearities are inherent to many interesting computational problems in real-world applications.

Here, we are interested in certified computation of *lower* bounds for roundoff errors. This is complementary of previous research [3], providing *upper* bounds.

Objectives The aim of this research work is two-fold:

1. reformulating the roundoff error problem as a generalized eigenvalue problem. The idea of this reformulation shall be based on the theoretical results from [2], relying on convex optimization (semidefinite programming, polynomial sums of squares).
2. implementing a tool solving this eigenvalue problem (e.g. with the `eigifp` Matlab toolbox) and comparing with recent competitive methods (e.g. `s3fp` [1]). The tool could possibly be included in the `Real2Float`[2] verification framework.

Required Skills

- Linear algebra basics
- Programming with C/C++/Matlab/OCaml according to preference of the candidate
- Convex optimisation, linear/geometric/semidefinite programming
- Related master courses: Software Development Tools and Methods, Efficient methods in optimization

A related PhD topic can be foreseen.

References

[1] W.-F. Chiang, G. Gopalakrishnan, Z. Rakamaric, and A. Solovyev. Efficient search for inputs causing high floating-point errors. In *Proceedings of the 19th ACM SIGPLAN Symposium on Principles and Practice of Parallel Programming*, PPoPP '14, pages 43–52, New York, NY, USA, 2014. ACM.

[2] J. B. Lasserre. A new look at nonnegativity on closed sets and polynomial optimization. 21(3):864–885, 2011.

[3] V. Magron, G. Constantinides, and A. Donaldson. Certified Roundoff Error Bounds Using Semidefinite Programming, 2015.

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[2] `http://nl-certify.forge.ocamlcore.org/real2float.html`