Time integration schemes for nonlinear diffusion equations: application to the coupling of oceanic and atmospheric boundary layers

Master of applied mathematics (research)   (*A PhD funding is foreseen*)

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Context

While climate change is causing many changes that affect our society, the numerical climate models assessed by the scientists in the framework of the IPCC (Intergovernmental Panel on Climate Change) are an indispensable tool for anticipating future risks. The development of the numerical models used in this context generally dissociates the dynamical kernel, which handles the resolved scales, from the physics, that account for under-resolved processes through so-called turbulent parameterizations (e.g. Gross et al., 2017). However, the dynamics and the physics are strongly connected. For example, in the oceanic and atmospheric boundary layers the usual physical parameterizations give rise to an additional nonlinear diffusion term which requires a special care to be integrated in time with sufficient accuracy and numerical stability (e.g. Wood et al., 2007; Nazari et al., 2013). In this internship we propose to work on the proper time integration of the boundary layer physics within ocean/atmosphere models.

Objectives

In this work we will consider a one-dimensional vertical diffusion equation where the diffusion coefficient depends nonlinearly on the diffused quantity to represent the turbulent mixing in the oceanic/atmospheric boundary layer. Due to the small vertical, relative to horizontal, grid resolution typically used in numerical models vertical terms impose a severe restriction on the time step if explicit-in-time methods are used (e.g. Lemarié et al. 2015) while simple standard implicit schemes may be prone to spurious numerical oscillations and severe loss of accuracy. The objective of the internship is to study alternative temporal discretizations of the nonlinear diffusion problem using, for example, the general framework of implicit Runge-Kutta schemes (e.g. Nazari et al., 2013). The stability properties of those schemes will be investigated and their benefits will be assessed using numerical experiments for specific scenarios for which reference solutions exist. Depending on the progress of this work, we could, then, study the extension of the proposed approach to the time integration of the coupling between oceanic and atmospheric boundary layers.
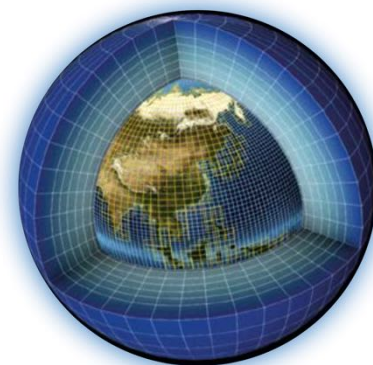
Prerequisites

- Basic knowledge in numerical analysis

- Programming skills in python and/or Fortran

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