Optimizing sums of Rayleigh quotients for model reduction

Research Context. The maximization (or the minimization) of a Rayleigh quotient is ubiquitous in linear algebra, notably because it is related to eigenvalue problems. Optimizing the sum of many Rayleigh quotients has recently emerged as a challenging problem with several applications in the multi-user MIMO system [3], sparse Fisher discriminant analysis in pattern recognition [5] and in nonlinear dimension reduction methods for reduced order modeling as in [6]. The problem consists in solving

$$\min_{x \in \mathbb{R}^n} \sum_{i=1}^{m} \frac{x^T A_i x}{x^T B_i x}.$$

where $A_i, B_i \in \mathbb{R}^{n \times n}$ are m symmetric matrices with nonnegative eigenvalues.

Goals. In [2], the author derives a framework to approximate from below the maximum of a polynomial. A natural research investigation track would be then to generalize this framework to the case of rational function sums. Practical experiments shall be performed through implementing a tool within the Matlab library `Gloptipoly3` [1] or the Julia library `MomentOpt`. One expected goal is to compare the performance of the tool with classical optimization algorithms, including (stochastic) gradient descent.

Working Context. The internship will be co-advised by Victor Magron (CNRS LAAS), Swann Marx (CNRS LS2N) and Olivier Zahm (INRIA Grenoble). The Master student will be hosted by the Mac team in the LAAS laboratory, located at Toulouse.

Required Skills. Motivated candidates should hold a Bachelor degree and have a solid background in **either** optimization, signal processing, control, real algebraic geometry or computer algebra. Good programming skills are also required. The candidates are kindly asked to send an e-mail with "M2 candidate" in the title, a CV and motivation letter to `vmagron@laas.fr`, `smarx@laas.fr` and `olivier.zahm@inria.fr`. Knowledge of French does not constitute a per-requisite.

A related PhD topic can be foreseen.

References

[1] D. Henrion, J.-B. Lasserre, and J. Lofberg. GloptiPoly 3: moments, optimization and semidefinite programming. *Optimization Methods and Software*, 24(4-5):761–779, 2009.

[2] J.B. Lasserre. Connecting optimization with spectral analysis of tri-diagonal Hankel matrices. 2019. `https://arxiv.org/abs/1907.09784`.

[3] G. Primolevo, O. Simeone, and U. Spagnolini. Towards a joint optimization of scheduling and beamforning for MIMO downlink. 2006. `http://www.lcs.poli.usp.br/isssta2006/isssta_cd/U1.pdf`.

[4] S. SY. Tan, A. Varvitsiotis, and V. YF. Tan. Analysis of Optimization Algorithms via Sum-of-Squares. 2019. `https://arxiv.org/abs/1906.04648`.

[5] M.C. Wu, L. Zhang, Z. Wang, D.C. Christiani, and X. Lin. Sparse linear discriminant analysis for simultaneous testing for the significance of a gene set/pathway and gene selection. 2009. `https://academic.oup.com/bioinformatics/article/25/9/1145/203375`.

[6] G. Zhang, J. Zhang, and J. Hinkle. Learning nonlinear level sets for dimensionality reduction in function approximation. 2019. `https://arxiv.org/abs/1902.10652`.