MSc thesis project "Simulation of time-inhomogeneous Stochastic Differential Equations with Local Time" (MSIAM2-Stat)

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Mention: research.

Phd: A PhD could be forseen.

Project objectives and required competences:

In this project we aim at exploring the question of the simulation of time-inhomogeneous one-dimensional Stochastic Differential Equations involving the Local Time of the unknown process (SDELTs). These equations are of the form

$$dX_t = \sigma(t, X_t)dW_t + b(t, X_t)dt + \beta(t)dL_t^\gamma(X) \qquad (1)$$

where the process $L^\gamma_\cdot(X)$ is the *Local Time* process of X on the time curve $t \mapsto \gamma(t)$. The local time is an almost surely continuous and increasing process. One of the possible ways to define it is through the limit

$$L_t^\gamma(X) = \lim_{\varepsilon \downarrow 0} \frac{1}{2\varepsilon} \int_0^t \mathbf{1}_{|X_s - \gamma(s)| \leq \varepsilon} d\langle X \rangle_s, \quad \forall t \geq 0.$$

The local time plays the role of a singularity in the dynamic (1), as it satisfies for example $dL_t^\gamma(X) = \mathbf{1}_{X_t = \gamma(t)} dL_t^\gamma(X)$.

The existence of solutions to (1) has been investigated only very recently (see [4]). Therefore the question of how to simulate the paths of X solution to (1) now arises.

One possibility should be to adapt the methodology proposed in [5] for the time-homogeneous case: in this work the authors apply a one-to-one transformation to the SDELT in order to get rid of the local time term, and then use an Euler scheme.

Another possibility could be provided by the explicit laws that are known about the time-Inhomogeneous Skew Brownian Motion (ISBM). The ISBM solves a very simple type of time-inhomogeneous SDELT, namely $dX_t = dW_t + \beta(t)dL_t^0(X)$. For example the explicit transition probability density of the ISBM is known (see [2]). One can wonder if this could help to produce *exact simulations* of the paths of the solution of (1), in the spirit of [1] (see also [3]).

Prerequisite is some knowledge of Stochastic Processes (and ideally of Stochastic calculus). Some knowledge of scientific computing is suitable (C/C++, Matlab,...).

References

[1] A. Beskos, O. Papaspiliopoulos, and G.O Roberts, *Retrospective exact simulation of diffusion sample paths with applications*, Bernoulli **12** (2006), no. 6, 1077–1098. MR 2274855 (2008c:65011)

[2] Pierre Étoré and Miguel Martinez, *On the existence of a time inhomogeneous skew brownian motion and some related laws*, Electron. J. Probab. **17** (2012), no. 19, 1–27.

[3] Pierre Étoré and Miguel Martinez, *Exact simulation of one-dimensional stochastic differential equations involving the local time at zero of the unknown process*, Monte Carlo Methods Appl. **19** (2013), no. 1, 41–71. MR 3039402

[4] Pierre Étoré and Miguel Martinez, *Time inhomogeneous Stochastic Differential Equations involving the local time of the unknown process, and associated parabolic operators*, to appear at Stochastic Processes and their Applications (SPA), 2017.

[5] Miguel Martinez and Denis Talay, *One-dimensional parabolic diffraction equations: pointwise estimates and discretization of related stochastic differential equations with weighted local times*, Electron. J. Probab. **17** (2012), no. 27, 1–30.