Research project

Absorbing boundary conditions for elasto-dynamics equations: mathematical analysis and numerical implementation

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- **Frame:** Full waveform inversion, a high resolution seismic imaging method, is based on the iterative minimization of the distance between observed and calculated data. The calculated data is obtained through the solution of the elastodynamics equations, which model the propagation of seismic waves within the subsurface. To accurately model the propagation of such waves, it is usual to consider the subsurface as a semi-infinite medium, with a free surface condition at the subsurface/air interface. Solving numerically these equations in a semi-infinite medium requires to design specific boundary conditions to preclude artificial reflections on the numerical boundaries of the domain. In practice, absorbing layers conditions, such as sponge layers (Cerjan et al., 1985) are combined with first-order absorbing boundary conditions (ABC) on the external edges of the domain (Engquist and Majda, 1977; Clayton and Engquist, 1977). Such ABC are generally designed for acoustic equations. The project aims at investigating first-order absorbing conditions for the general elastodynamics equations.

- **Project:** The first step of the project will be the mathematical derivation of these absorbing boundary conditions using a systematic approach based on the analysis of the elastodynamics equations as first-order hyperbolic system. After a symmetrization of these equations, it becomes possible to isolate the different components of the wavefield traveling at different speeds in the different directions of space. This is done through the definition of projectors on eigenspaces associated with the symmetrized wave equation symbol. Using these projectors, it becomes easy to define first-order absorbing boundary conditions on each border of the medium by canceling the component of the wavefield traveling back to the computational domain. We will explore this systematic formulation starting from 2D VTI elastodynamics equations and possibly up to the general 3D elastodynamics equations. We will compare it with the state-of-the art approach of Lysmer and Kuhlemeyer (1969). We will then implement the resulting 2D VTI absorbing boundary conditions in a finite-difference staggered grid code which will be made available to the student. If successful, this could be also implemented in a 2D spectral element based modeling code, as a first step towards its integration into the 3D spectral element code SEM46, used for full waveform modeling and inversion within the SEISCOPE project (Trinh et al., 2019).

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- **Competences:** Background in numerical analysis, seismic modeling and imaging; willingness to be involved in mathematical and numerical developments

References

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