Game-theoretical analysis of deep neural networks

Ph.D. thesis proposal

Hubert Curien and GATE laboratories open a Ph.D. position to address open questions related to the theoretical understanding of deep neural networks.

Keywords Deep neural networks, game theory, optimization.

1 Context

In recent years, deep learning has imposed itself as the state of the art ML method in many real-world tasks, such as computer vision or natural language processing to name a few [1]. While achieving impressive performance in practice, training DNNs requires optimizing a non-convex non-concave objective function even in the case of linear activation functions and can potentially lead to local minima that are arbitrary far from global minimum. This, however, is not the typical behaviour observed in practice, as several works [2, 3] showed empirically that even in the case of training the state-of-the-art convolutional or fully-connected feedforward neural networks one does not converge to suboptimal local minima. Such a mysterious behaviour made studying the loss surface of DNNs and characterizing their local minima one of the topics of high scientific importance for the ML community.

2 Scientific project

In order to provide novel insights into the behaviour of DNNs, our goal will be to study them as instances of congestion games [4], a class of games often used to model network traffic and communications. This particular choice is due, on one hand, to the fact that both DNNs and congestion games can be modeled as direct acyclic graphs (DAGs), while, on the other, congestion games are arguably among the most studied classes of games in GT that are known to exhibit many attractive properties. The approximate objectives of the Ph.D. thesis in this context willconsist in:

1. Proposing a general approach of finding a congestion game equivalent to a given DNN.
2. Translating the different quantities of interest often analyzed in the context of congestion games to DNNs in order to provide a novel theoretical analysis for them.
3. Using extension theorems [5] to study the speed of convergence of online optimization strategies when applied to DNNs.

Some preliminary encouraging results obtained for such approach have been obtained recently by supervisors in [6] and the Ph.D. candidate is expected to address the open problems mentioned in the latter paper.

3 Expected skills

Ideal candidates will have a strong background in both **machine learning** and **game theory**, but anyone with a Master's degree in **applied/pure mathematics** is encouraged to apply. Proficiency in at least one programming language commonly used in machine learning community would be a plus.

4 Application

- Send your CV, motivation letter, and academic records to
  - `ievgen.redko@univ-st-etienne.fr`
  - `charlotte.laclau@univ-st-etienne.fr`
- **Salary:** About 1700€ per month.
- **Location:** The successful candidate will be a member of Data Intelligence team, within the Hubert Curien laboratory situated in Saint-Etienne, France.
- **When:** The successful candidate shall start in September, 2021.

References

[1] Ian Goodfellow, Yoshua Bengio, and Aaron Courville. *Deep Learning*. MIT Press, 2016.

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[3] Ian J. Goodfellow and Oriol Vinyals. Qualitatively characterizing neural network optimization problems. In *ICLR*, 2015.

[4] Robert W. Rosenthal. A class of games possessing pure-strategy nash equilibria. *International Journal of Game Theory*, 2:65–67, 1973.

[5] Tim Roughgarden. Intrinsic robustness of the price of anarchy. *ACM*, 62(5), 2015.

[6] Nina Vesseron, Ievgen Redko, and Charlotte Laclau. Deep neural networks are congestion games: From loss landscape to wardrop equilibrium and beyond. *AISTATS*, 2021.